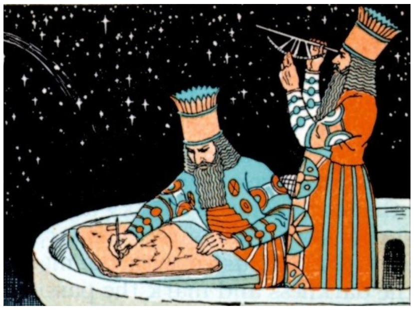
Saturday Morning Astrophysics at Purdue

Kepler's Laws of Planetary Motion

Introduction

For centuries, humans gazed up at the skies and, guided by their creativity and imagination, speculated about what they saw, attempting to explain their observations. Among these pioneers of astronomy are the perhaps familiar names of Aristarchus (3rd century BC), Ptolemy (100-170), Copernicus (1473-1543), Brahe (1546-1601) and Johannes Kepler (1571- ?).



www.astronomytrek.com/10-top-astronomers-from-the-ancient-world/

Like many philosophers of his era, Johannes Kepler had a mystical belief that the circle was the Universe's perfect shape. As a manifestation of Divine order, Kepler struggled for many years to make sense of planetary motion using circular orbits. Applying mathematics to observational data, he eventually unlocked the answer.

In the following activities, you will be able to collect and analyze your own data to unlock the same secrets that eluded Kepler.

Before you begin

Watch Professor Kepler explain the scenario and introduce the lesson; https://bit.ly/3l21FEl

Materials

Orbital diagram
Small ruler
String (20 cm)

Calculator
Pencil
Excel or other spreadsheet (optional)

Part 1 - Collecting Your Data

The diagram accompanying this procedure shows the path of a body in its orbit around the Sun. The numbers, which count counter-clockwise, identify positions at equal intervals of time.

Procedure

1. Measure the distance between the Sun and the orbiting body (for example the dwarf planet Pluto) at each numbered position to the nearest 0.1 cm. Record these distances in **Table 1**. Then convert these distances from cm to Astronomical Units (AUs) by multiplying your measurements by the conversion factor on the orbital diagram, and enter your answers in **Table 1**. For example, if you measure the distance as 7.4 cm,

   $$7.4 \text{ cm} \times 4.42 \text{ AU/cm} = 32.7 \text{ AU}$$

   (Note that 1 cm = 4.42 AU is the conversion value from the orbital diagram for Pluto.)

2. Next, measure the **distance along the curve** of the orbit that the orbiting body travels during each one-year interval. (Mark the distance on a string, then measure the string with a ruler.) Record these distances to the nearest 0.1 cm in **Table 2**.

   Convert these lengths from cm into miles (or km) in space, using the conversion units shown at the bottom of the orbital diagram. Record these values that represent the distance the body actually travels in space in each sector of the orbital period in **Table 2**.

3. Then, calculate the **average speed** during each time sector by dividing the distance by the number of hours in a year. Record these values in **Table 2**.

   To get you started…,
   First, calculate the number of hours in one year:

4. Calculate the speed (distance over time) for each time interval, and enter those values in the **Average Speed** column **Table 2**.

5. Finally, the imaginary lines between Sun and the orbiting body divide the orbital area into equal intervals of time. A simple way to estimate the area of any given time interval is to count the number of squares in that sector. Determine the areas of the intervals which you are assigned and record these values in **Table 2**.

   (As a guide, if the square is ½ or more contained in the sector, then count it, it if is less than ½, then do not include it.

TABLE 1 – Sun to Orbit Distance

Sector position	**Sun-to-orbit distance** (cm on paper)	**Sun-to-orbit distance** (in AU)
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

TABLE 2 – Average Orbit Speed

Area swept from…	**Distance traveled (1) (cm)**	**Distance traveled (2) (in miles)**	**Average speed (3) (mi/h)**	**Area swept (# of squares)**
0 –1				
1 –2				
2 – 3				
3 – 4				
4 – 5				
5 – 6				
6 – 7				
7 – 8				
8 – 9				
9 – 10				
10 – 11				
11 – 0 (12)				

(1) Distance along each arc (measured with string and ruler).

(2) Multiply distance traveled (in cm) by the conversion factor on the bottom of the orbital diagram.

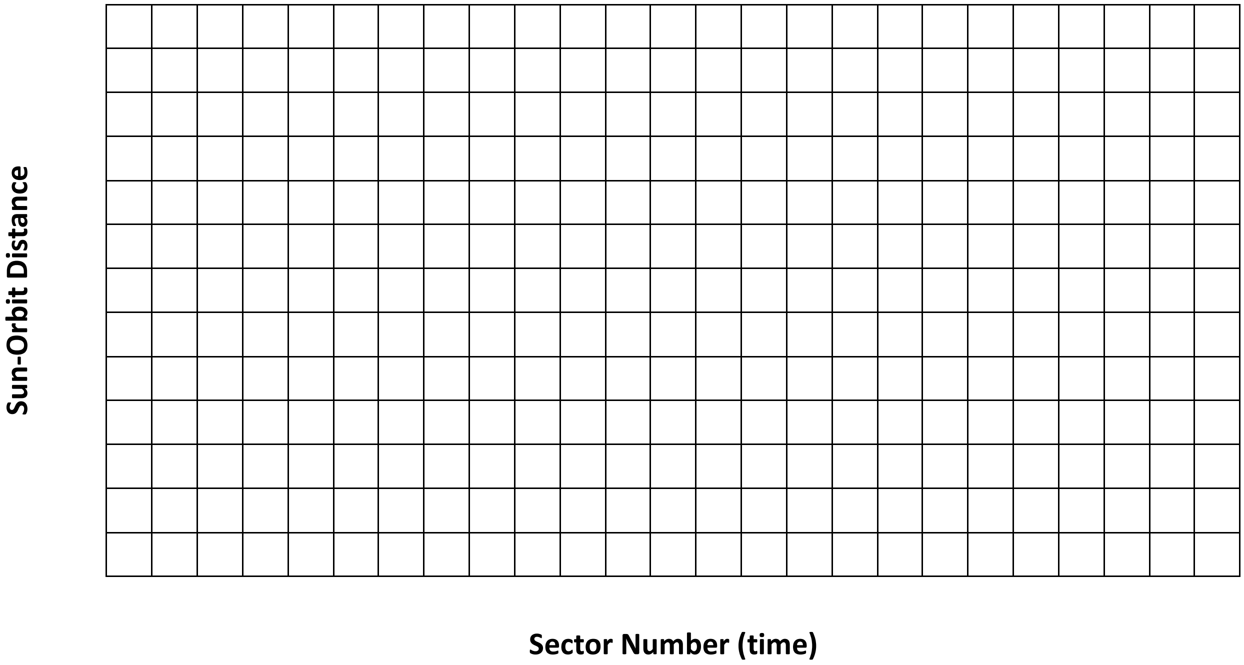
(3) Divide distance (mi or km) by hours in the year.

Part 2 - Graphing Your Data

Now, let's try to make sense of your data. Plot each of these three data sets on separate graphs, either using the templates below or an electronic spreadsheet. Create and number a suitable scale for the axes in each graph.

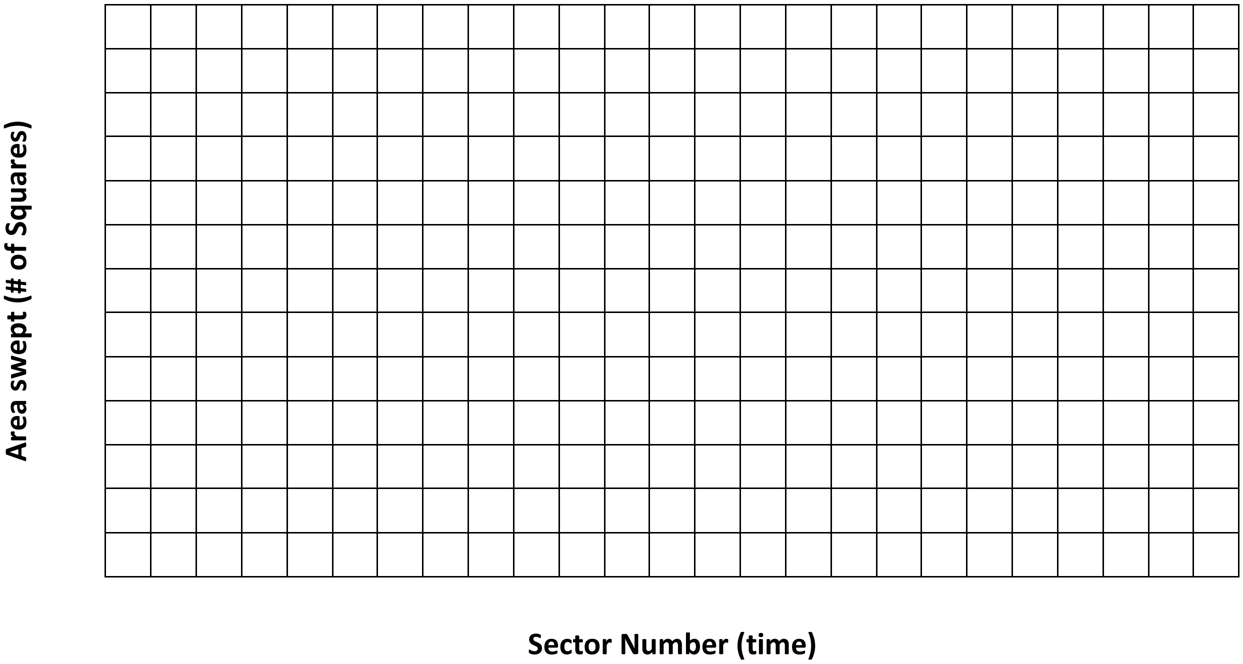
1. Plot the **sun-to-orbit distances** (from Table 1 – in AU) vs. the **sector number** (time). Connect the dots with a best-fit curve.

Graph 1 – Sun to Orbit Distance versus Year



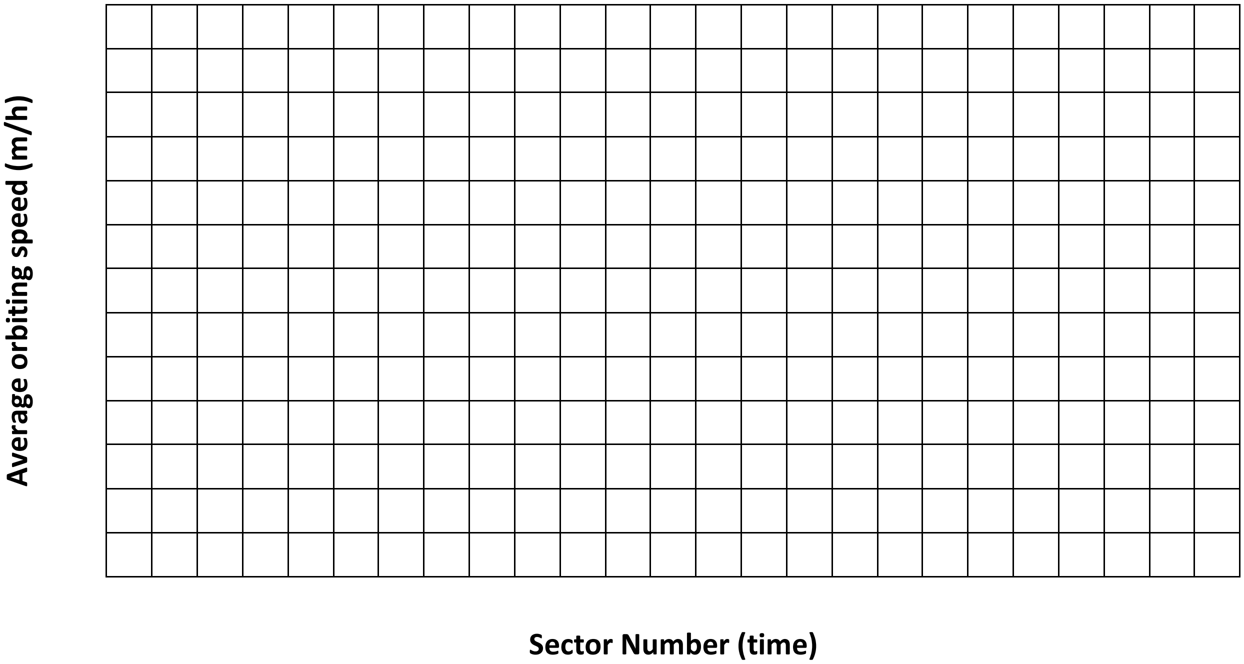
2. Now plot the **area swept** by the imaginary line (the line connecting the Sun and orbiting body) vs. the **sector number**. Smoothly connect the dots with a curve.

Graph 2 - Area Swept versus Time interval



3. On graph 3, plot the **average speed** of the orbiting body (in either miles or km per hour) vs. the **sector number**. You will need to determine your own scale for the y-axis.
    a. Connect the dots with a smooth curve.
    b. Identify the **perihelion** (position when the body is closest to Sun) and **aphelion** (when the body is furthest from Sun) on your graph.

Graph 3 - Average orbiting Speed versus Year



QUESTIONS

1. What is the maximum distance of the orbiting body from the Sun (i.e. aphelion) in AU?

   ____

2. What is the minimum distance from the Sun (i.e., the perihelion) in AU?

   ____

3. Calculate the orbiting object's average distance from the Sun, by averaging the aphelion and perihelion distances.

   ____

4. Looking at your graphs, describe the relationship do you observe between the orbiting object's **distance from the Sun** and the **speed** at which it is moving?

5. Looking at you graphs, describe the relationship do you observe between the **distance from the Sun** and the **area swept** out in each sector of the orbital diagram.

6. Through trial and error, you might be able to discover the mathematical relationship between the **time** it takes a planet to orbit around the Sun (defined as the **period = P**) and the average **distance** away the planet is from the Sun (defined as **D**). It is a power law of the form,

$$\mathbf{P^x = D^y}$$

where x and y are whole numbers (e.g., 1, 2, 3, etc.).

If you think you have an answer, try your formula with these planet data:

Planet	Period (in days)	Average distance (in AU)
Mercury	88.0	0.39
Venus	224.7	0.72
Earth	365.25	1.00
Mars	687.0	1.52
Jupiter	4333	5.20
Saturn	10832	9.52
Uranus	30707	19.1

What's your answer? Write what you think the correct form of the law is below.

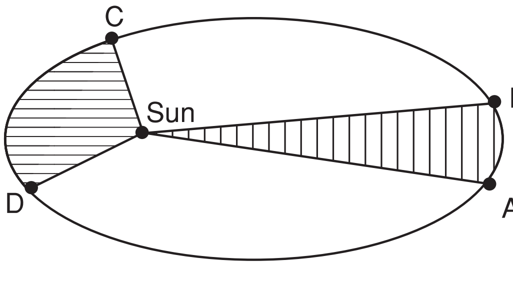
"I much prefer the sharpest criticism of a single intelligent man, to the thoughtless approval of the masses." Johannes Kepler

Kepler’s Three Laws of Planetary Motion
(Teacher Background)

Tycho Brahe (1546-1601) was a Danish astronomer who dedicated his life to recording the positions of planets more precisely than anyone had ever achieved before. In recognition for his work, the king of Denmark gave Tycho an island and a small fortune to build an observatory. (Rumor has it that it was 10% of what the country made in a year!) He used this to build the world’s most accurate instruments for measuring astronomical positions. Brahe worked under the idea that the Earth was at rest, and that the Sun and planets orbited around it.

Johannes Kepler (1571-1630), also from Denmark, did not share the same belief as Brahe. Instead, Kepler believed that the Sun was at the center of the solar system and that the planets orbited around in perfect circles, just as Nicolaus Copernicus had outlined in 1543. Although observations at that time suggested otherwise, Kepler was convinced that the uncertainties in measurements were large enough that the Copernicus picture was correct. Kepler realized that only Brahe’s measurements were of sufficient precision to settle the question and went to work for him in 1600. Tycho was nervous about being disproven and withheld vast quantities of data from Kepler even though they worked together. It wasn’t until Tycho died the following year in 1601 that Kepler was able to access (or, as he admits, “steal”) the observations. Kepler worked with the data for nine years before accepting that his geometric scheme was actually wrong. In its place, he formulated three laws of planetary motion. Kepler’s Laws (as we refer to them today) are obeyed throughout the universe. Planets, comets, and asteroids that orbit the Sun all follow these laws. The moon and man-made satellites orbiting the Earth also follow these laws.

1) Objects orbiting the sun (planets, asteroids, comets, etc.) move in **elliptical** orbits with the Sun at a focus.
2) In their orbits around the sun, the objects sweep out equal areas in equal times.



The area of A-B-Sun equals the area of C-D-Sun. Therefore, the planet is moving slower going from A to B and faster when going from C to D.

The square of the time to complete one orbit (i.e., the period P) is proportional to the cube of the average distance D from the sun. If P is in years, and D is in Astronomical Units (AU) we can write this as

$$\mathbf{P^2 = D^3}$$